5BL Lab 10 Report

Alex Nava, Nolan Cheng

2 April 2026

Contents

Introduction

RLC circuits are used heavily in electrical engineering, PCB engineering, and a whole host of other fields, and as a result understanding the theoretical models and ensuring they are correct is imperative. That's why in this lab we analyzed the behavior of both RC and RLC circuits, using basic circuit components, an oscilloscope, a function generator, and a homemade capacitor. We compared the data we collected to theory in order to determine whether or not they agree.

Theory

RC circuits exhibit charging behavior that can be found as follows:

$$V = IR,$$

$$V = \frac{Q}{C},$$

$$\frac{dQ}{dt} = -\frac{1}{RC}Q,$$

$$Q(t) = CV_0e^{-\frac{t}{RC}},$$

$$V(t) = V_0(1 - e^{-\frac{t}{RC}}).$$

Similarly, for the discharging of a capacitor,

$$V(t) = V_0e^{-\frac{t}{RC}}.$$

Using these equations, we can measure the voltage across RC circuits over time, and the fit the data in order to experimentally determine the time constant, $\tau = RC$. Similarly, we can also use this method to determine the capacitance or resistance of a component given we know the value of the other. We will use this fact to determine the capacitance of a homemade capacitor.

RLC circuits can typically analyzed using these equations:

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{CL} = V_0 \tag{1}$$

Where q is the charge, t is time, R is the equivalent resistance, L the equivalent inductance, and C the equivalent capacitance.

And differentiating with respect to time results in the equation:

$$\frac{d^2 I}{dt^2} + \frac{R}{L}\frac{dI}{dt} + \frac{I}{CL} = 0 \tag{2}$$

This equation is similar to the equation governing damped harmonic oscillators and thus can also be over-, under-, and critically damped. In this part we have two different cycles and we attempt to find the type of oscillation each follows.

Furthermore, using the Ansatz I = e^{rt} results in the quadratic equation:

$$r = \frac{-\frac{R}{L} \pm \sqrt{\frac{R}{L}^2 - \frac{4}{RL}}}{2} \tag{3}$$

And depending on the values of R, C, and L can result in different equations for I, corresponding to under damped, over damped, and critically damped. Notice that these are similar to terms and equations of a damped harmonic oscillator. And for a circuit that is missing one or more of those components we set the values of the respective variable to 0, so for example an RC circuit that is composed of resistor(s) and capacitor(s) the system can be analyzed by setting L to 0.

In this lab we have both an RLC and RC circuits which have equations:

RL: - Discharging:

$$V_c(t) = V_0 e^{-t/\tau} \tag{4}$$

- Charging:

$$V_c(t) = V_0 \left(1 - e^{-t/\tau}\right) \tag{5}$$

RLC: - Current:

$$I = e^{-\alpha t}(Ae^{bt} + Be^{-bt}) \tag{6}$$

- Voltage:

$$\frac{V}{V_0} = A * (e^{-d*t}) \tag{7}$$

Note that for any data analysis, the following equation can be used to determine whether or not two values agree, where α is their respective errors:

$$|x - y| \leq 2\sqrt{\alpha_x^2 + \alpha_y^2}.$$

If this inequality holds true, then the values agree.

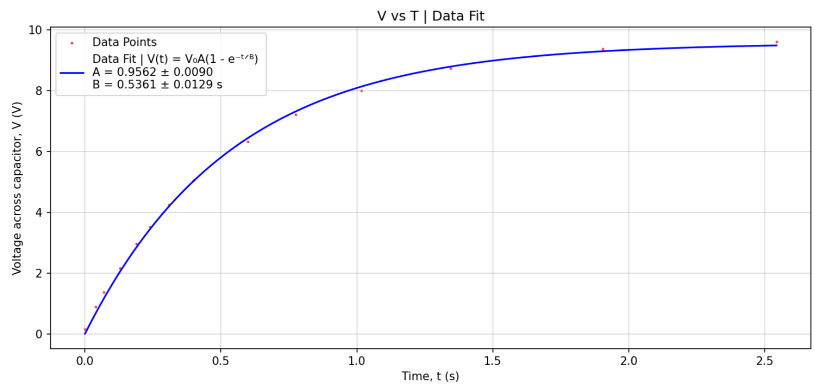

Figure 1: Charging data for an RC circuit in Experiment 1A

1 Experiment 1

In this experiment, we will be using the oscilloscope to experimentally test an RC circuit. First, we will attempt to determine the time constant of an RC circuit given a resistance of 10 $k\Omega$ and 47 μF. Second, we will create our own capacitor using aluminum foil, paper, and tape, and then attempt to determine its capacitance given its measured charging or discharging behavior.

For the first trial when attempting to determine the RC constant, we will use the charging behavior,

$$V_C(t) = V_0 A(1 - e^{-\frac{t}{B}}),$$

where A is the amplitude and B is the time constant RC.

For the second trial when attempting to determine the RC constant, we will use the same charging behavior as mentioned, as well as the discharging behavior

$$V_C(t) = V_0 A e^{-\frac{t}{B}}.$$

In order to determine whether our measured values agree with theory, we can simply perform the agreement test.

1.1 Experiment 1A: Measuring the RC Time Constant

For this experiment, we will use a $10k\Omega$ resistor and a $47\mu F$ capacitor. By connecting them in series alone with our AC current source, by measuring the charging behavior of the capacitor using an oscilloscope, we should find that

$$V_C(t) = V_0 A(1 - e^{-\frac{t}{B}}).$$

This was done by wiring the resistor and capacitor in series, with each end connected to a function generator. This was then also fed into an oscilloscope, which read both the function generator and the voltage across the capacitor.

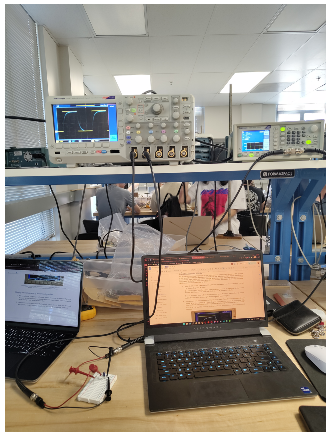

Figure 2: Experimental setup of experiment 1A

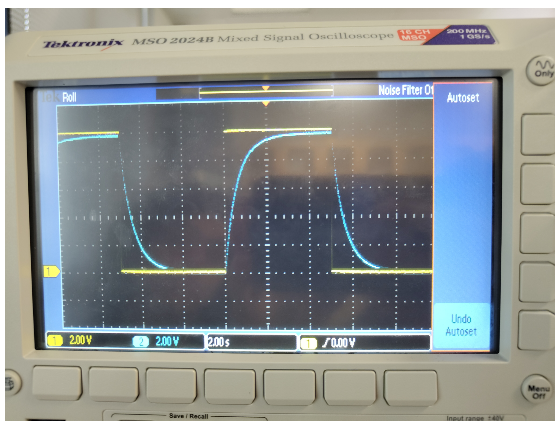

Figure 3: Charging and discharging behavior of the RC Circuit

By freezing the frame as the oscilloscope is charging, we can use the cursors on the oscilloscope to find the voltage at particular times. The setup for this can be seen in Figure 2.

After turning our function generator on and freezing the oscilloscope, we obtain the following as shown in Figure 3. By decreasing the time intervals, we were able to zoom in on the charging behavior and from there, measured our data.

After measuring our charging data, we find the data in Figure 1. Given that B is our time constant, we find that it is $0.5361 \pm 0.0129s$. Theoretically given our circuit, our time constant should be $RC = (10k\Omega)(47\mu F) = 0.47s$. Using propagated uncertainty, given that the uncertainty of our resistor is 1% or 100Ω

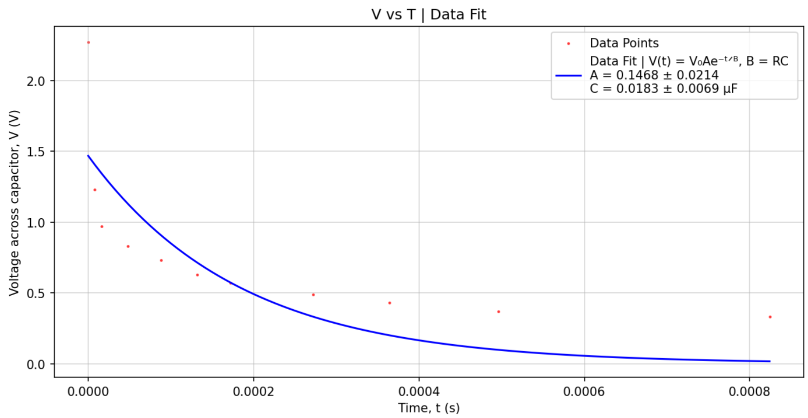

Figure 4: Discharging data for an RC circuit in Experiment 1B

and the uncertainty of our capacitor is 5% or $\pm 2.35\mu F$,

$$\alpha_{RC} = \sqrt{(100 * .000047)^2 + (.00000235 * 10000)^2} s,$$

$$\alpha_{RC} = 0.024s.$$

Therefore our theoretical RC time constant is $RC = 0.470 \pm 0.024s$. Performing the agreement test with our experimentally determined value,

$$|0.5361 - 0.47| \leq 2\sqrt{0.024^2 + 0.0129^2},$$

$$0.0661 \leq 0.0545.$$

Given that the inequality does not hold true, the values do not agree with one another. As our data fits well, this is likely due to idealizations of our circuit. Since our actual circuit also relies on the internal components of both the function generator and oscilloscope, it is likely that we have other resistances that are impacting our measurements.

1.2 Experiment 1B: Handcrafting a Capacitor

For this experiment, we used a $10k\Omega$ resistor in series with the capacitor that we made. In theory, the capacitance of our capacitor is given by

$$C = \epsilon_r \epsilon_0 \frac{A}{d},$$

where A is the area of the plates, d is the separation distance between the plates, and ϵ_r is the dielectric constant which is roughly 3.8 for paper. This gives a theoretical capacitance of

$$C = 3.8(8.854 \times 10^{-12})(.2159 \times 279.4)(0.0001)^{-1},$$

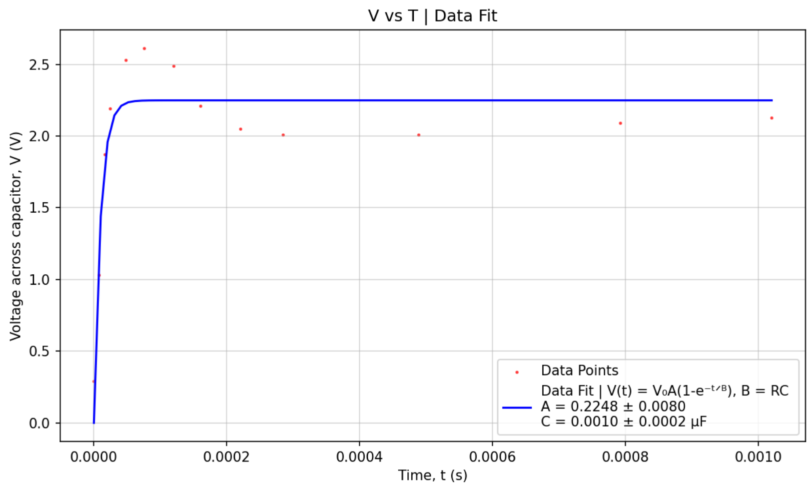

Figure 5: Charging data for an RC circuit in Experiment 1B

$$C = 2.02 \times 10^{-8} F = 20.2 nF$$

Our experimental setup is identical to that of Experiment 1A, but instead just uses our homemade capacitor. This setup can be seen in Figure 6. From this setup, after turning the function generator on, we were able to obtain the graph of our charging and discharging data as shown in Figure 7. The extreme irregularity of our charging data is obvious here.

After fitting our data as shown in Figure 4, we can see that we find our capacitance is roughly $0.0183 \pm 0.0069 \mu F = 18.3 \pm 6.9 \mu F$ when fitted to our discharging data. When fitted to our charging data as shown in Figure 5 we get a capacitance of roughly $0.0010 \pm 0.0002 \mu F = 1.0 \pm 0.2 nF$.

The capacitance for our charging data does not come close to our theoretical value, but this is to be expected; By simply by looking at our data, we can see that our charging behavior is incredibly irregular. We are uncertain of why our charging data exhibits such strange behavior while our discharging data at least constantly discharges.

That being said, our discharging fit data is still very skewed. Our calculated best fit line does not follow the path of our data well, likely do our capacitor's unique discharging behavior. This can be seen in the fact that it never fully discharges but instead plateaus around 0.33 V and also has an extreme voltage drop at the start.

Collectively, this means that our capacitor does not follow the theoretical behavior of an RC circuit. Despite this, when discharging, our calculated capacitance does still agree with our theoretical value. Note: - We attempted this with a capacitor using tape instead of glue and got a far closer value to the theoretical one, but we had insofficient time to recollect all our data.

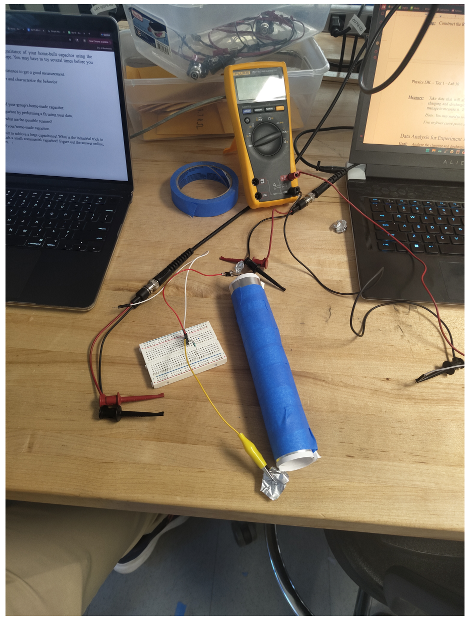

Figure 6: Experimental setup of Experiment 1B

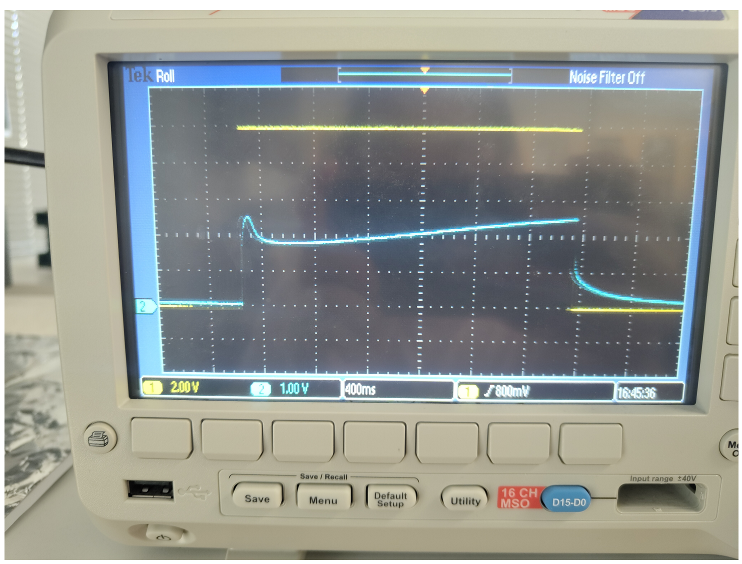

Figure 7: Charging and discharging data for Experiment 1B

2 Experiment 2

In this section, we focused on RLC circuits which can be analyzed using this equation:

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{CL} = V_0 \tag{8}$$

Where q is the charge, t is time, R is the equivalent resistant, L the equivalent inductance, and C the equivalent capacitance.

And differentiating with respect to time results in the equation:

$$\frac{d^2I}{dt^2} + \frac{R}{L}\frac{dI}{dt} + \frac{I}{CL} = 0 \tag{9}$$

This equation is similar to the equation governing damped harmonic oscillators and thus can also be over-, under-, and critically damped. In this part we have two different cycles and we attempt to find the type of oscillation each follows.

Furthermore, using the Ansatz I = e^{rt} results in the quadratic equation:

$$r = \frac{-\frac{R}{L} \pm \sqrt{\frac{R}{L}^2 - \frac{4}{RL}}}{2} \tag{10}$$

And depending on the values of R, C, and L can result in different equations for I, corresponding to under damped, over damped, and critically damped.

2.1 2A

Here we looked at a 100 mH inductor, 0.1 µF capacitor, and a 50 Ω resistor all in series (in this case the 50Ω was created using two 100Ω resistors in parallel).

using the equation above yields the result:

$$I(t) = e^{-\alpha t}\left(A\cos(\omega t) + B\sin(\omega t)\right) \tag{11}$$

Which can be used to find:

$$V(t) = \tilde{A}e^{-\alpha t}\cos(\omega t - \phi) \tag{12}$$

where $\omega = \sqrt{\omega_0^2 - \alpha^2}$, $\omega_0 = \frac{1}{\sqrt{LC}}$, $\alpha = \frac{R}{2L}$, and b=$\sqrt{\alpha^2 - \omega_0^2}$, and A and B are set by initial conditions.

The fit yielded the parameters $\alpha = 8.72 \times 10^2\,\mathrm{s}^{-1}$, $\omega = -9.85 * 10^3\,s^{-1}$, $\phi = 8.33 * 10^1\,rad$, and $A = 2.03 * 10^{-1}$.

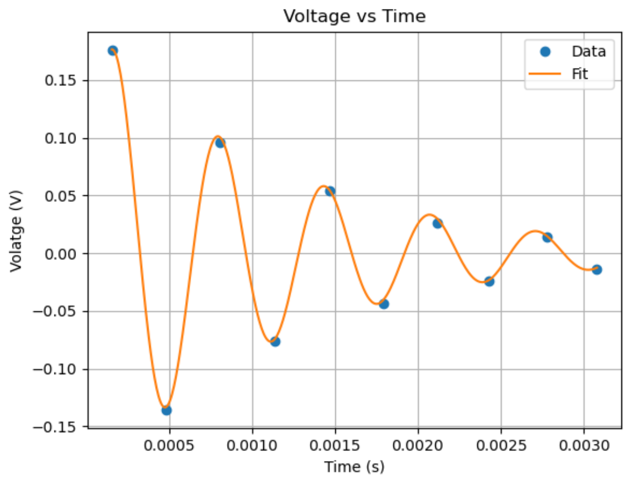

Figure 8: Voltage (V) vs Time (s) for the RLC circuit of 2A including both the raw data in blue and the fit using the theoretical under damped model in orange

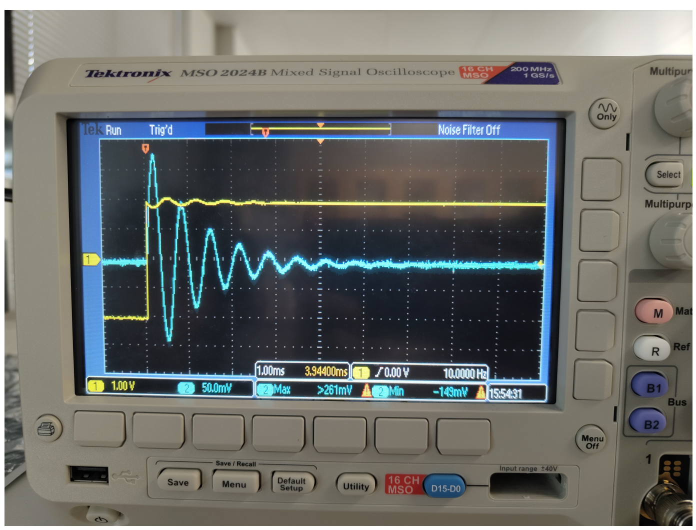

Figure 9: Raw data from lab, the yellow is the supplied voltage and the blue is the voltage from the start of our circuit to the end

Furthermore, the theoretical values for each variable was: $\alpha \approx 250\frac{1}{s}$ $\omega \approx 10000\frac{1}{s}$

And comparing them shows:

$3.488\omega_{th} = \omega_{ex}$

Which has an agreement of about 1.19, meaning they agree

$0.985\alpha_{th} = \alpha_{ex}$ Which has an agreement of about 6.5 indicating they don't agree

2.2 2B

Here we used a 100 mH inductor, 47 µF capacitor, and a 1 $k\Omega$ resistor, notice that this is a similar set up as the previous circuit but with more capacitance and vastly more resistance.

Plugging those values into the quadratic and differential equations, results in the theoretical model,

$$I = e^{-\alpha t}(Ae^{bt} + Be^{-bt}) \tag{13}$$

where $\alpha = \frac{R}{2L}$, b=$\sqrt{\alpha^2 - \omega_0^2}$, and A and B are set by initial conditions.

Furthermore using the fact that V = IR using the previous equation yields that:

$$V = Re^{-\alpha t}(Ae^{bt} + Be^{-bt}) \tag{14}$$

Which can also be expressed as

$$\frac{V}{V_0} = A * (e^{-d*t}) \tag{15}$$

Where V_0 is the provided voltage and d $\approx \alpha - b$. Plugging in the values of each component we can find each value:

$\alpha = 5000$ $\omega_0 \approx 461.2$ $b \approx 4978.7$ $d \approx 21.3\frac{1}{s}$ A = $\frac{V(0)}{V_0} \approx 0.44$ (We did not get the data for t=0, so instead this is for t=156μs

experimentally, we found values for the voltage at times corresponding to the peaks and valleys of the circuit discharging.

The fit yielded values of: A = 14.85939931 d = 6.03232346 $*10^{-3}$

Compared to theoretical values: $A_{ex} \approx 33.77 A_{th}$ $d_{ex} \approx 2.83 * 10^{-4} d_{th}$

Which clearly does not agree (using a proper agreement test gives around 40000, which is far too high to agree and thus disagrees, and A it gives a value of 0.614). But keep in mind for A's theoretical value we did not use t=0 and thus gives a different value than it should, and because A was wrong that would effect the value for d as well. Additionally the $\frac{V}{V_0}$ equation is only an approximation so it is likely these errors compiled and caused the final values to be vastly different.

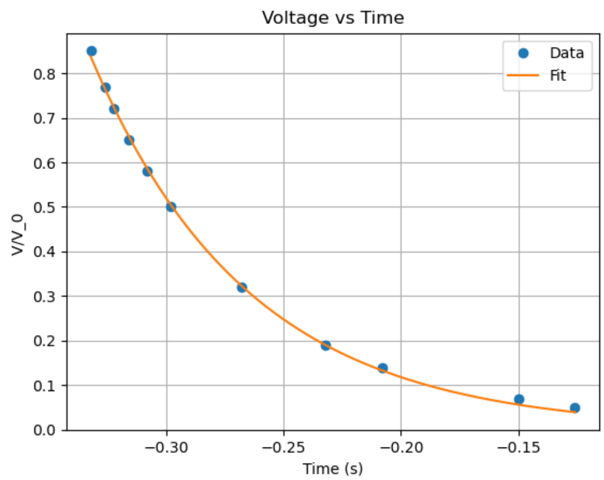

Figure 10: Voltage (V) vs Time (s) for the RLC circuit of 2B including both the raw data in blue and the fit using the theoretical over damped model in orange

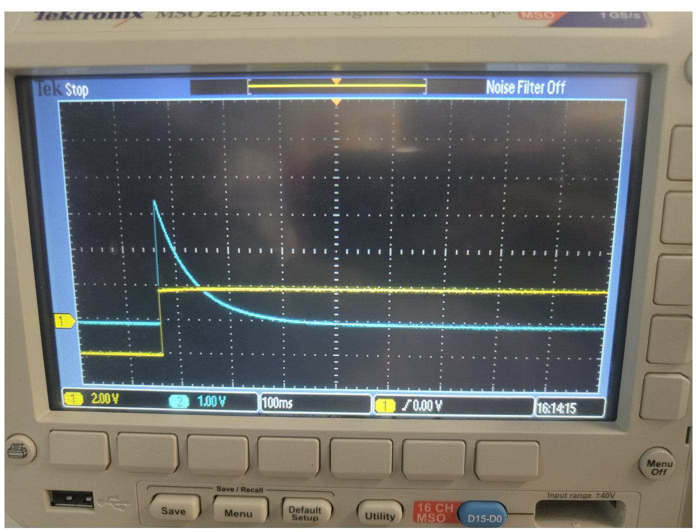

Figure 11: Data from lab, yellow is supplied voltage and blue is the voltage of the circuit, notice that it resembles the fitted graph

3 Conclusion

1: For 1A we didn't account for attributes of the internal components, such properties of the function generator and oscilloscope, and as an improvement we will ensure to include that in future data collections. Whereas for 1B, most of our error likely formed from the resistor used. For one, we had far greater capacitance than expected likely due to the use of glue instead of tape (which would have used instead now knowing how much it altered the data). Also, we believe the function generator creates square waves, which we used, by using Fourier series which always contain the Gibbs phenomenon which could explain why our data is more curved than the theoretical model. As a result our theoretical model differed from the actual data.

2: We found that the values for ω agree and α disagree with the theoretical under damped model, where as for the over damped model both A and d don't agree. This could have been improved by having a value for t=0. Furthermore, the resistance of things like the wires and bread board were unaccounted for, and even though they have values significantly smaller than that of the resistance provided in both cases, the theoretical model could have more accurately fit the data.

While we didn't have time to implement these changes due to our limited class period, if we were to do it again we would hope to adopt new tactics.

Group Contributions

All data collection was done by both Alex Nava and Nolan Cheng in conjunction. All of the data analysis and writing for Experiment 1 was done by Nolan Cheng, before being review by Alex Nava. All of the data analysis and writing for Experiment 2 as well as the conclusion was done by Alex Nava, before being reviewed by Nolan Cheng. The introduction was written in part by both Alex Nava and Nolan Cheng.